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Exhaustification and Presupposition: Argument for Weak Negation

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Abstract. This study investigates the two definitions of exhaustification discussed by Spector & Sudo (2017). It is observed that strengthening inclusive disjunction to exclusive disjunction via exhaustification operator creates a non-trivial prediction on presupposition projection. I argue, contra Spector & Sudo (2017), that the exhaustification operator employs *Weak Negation*, which cancels the presupposition of its prejacent.

1 Background

This study investigates an interplay between *presupposition projection* and *strenthening via exhaustification* in disjunction. Throughout this study, I notate atomic propositions as $p, q, r \ldots$, and (atomic or non-atomic) propositions as $\phi, \psi, \chi, \ldots$, and (inclusive) disjunction as $\phi \vee \psi$. For the ease of exposition, I assume a presupposition of sentences is always represented as an atomic proposition. I notate a proposition ϕ with a presupposition p as ϕ_p. The primary concern of this study is disjunctions in the form $\phi \vee \psi_p$.

Karttunen (1973) observes that the presupposition p of $\phi \vee \psi_p$ becomes the presupposition of the entire disjunction (i.e., the presupposition *projects*) except when $\neg\phi$ entails p. Consider the contrast in (1). The use of *stop* in ψ induces the presupposition that [$_p$ *John used to run regularly*]. This presupposition p is projected in (1a), where $\neg\phi$ does not entail p. In (1b), $\neg\phi$ does entail p and the disjunction does not have any presupposition. Borrowing the terminology from Karttunen (1973), I say the presupposition p is *filtered* in disjunction $\phi \vee \psi_p$ if $\neg\phi$ entails p.

(1) a. Either [$_\phi$ John hates exercise] or [$_\psi$ he **stopped** running regularly].
Presuppose: [$_p$ John used to run regularly]

b. Either [$_\phi$ John never used to run regularly] or [$_\psi$ he **stopped** running regularly].
No presupposition

The filtering effect is often analyzed with the *Strong Kleene* definition of disjunction, which is characterized as having the truth table in (2).

(2) Strong Kleene Disjunction

$\vee$	1	0	$*$
1	1	1	1
0	1	0	$*$
$*$	1	$*$	$*$

The Strong Kleene logic is a trivalent logic that has three values for propositions: 1 for *true*, 0 for *false*, and $*$ for *undefined*. I assume that a proposition receives the $*$ value only when its presupposition is not true. Under the Strong Kleene logic, the value $*$ can be conceptualized as *unknownness*. Disjunction $\phi \vee \psi$ receives $*$ when its truth value (1 or 0) is unknown or undetermined given the values of the disjuncts. For instance, when ϕ is 0 and ψ is $*$ the value of disjunction $\phi \vee \psi$ is $*$, because the disjunction might be true (if ψ turns out to be 1) or false (if ψ turns out to be 0).

A proposition ϕ presupposes p if the truth of p is a prerequisite for ϕ to be 1 or 0. Then under the definition in (2), $\phi \vee \psi_p$ presupposes $\phi \vee p$ (equivalently $\neg\phi \rightarrow p$).

(3) Presupposition of disjunction under (2)
$\phi \vee \psi_p$ presupposes $\neg\phi \rightarrow p$

Consider the case in (1b) where the presupposition is filtered. The sentence is predicted to presuppose that $\neg\phi \rightarrow p$: *if* [$_{\neg\phi}$ *John used to run*], [$_\psi$ *he used to run*]. But this presupposition is a tautology, and its truth is guaranteed in any context/worlds. Therefore, the sentence has no (non-trivial) presupposition, as expected.

However, the Strong Kleene disjunction makes a wrong prediction for (1a). Recall that intuitively (1a) presuppose that [$_p$ *John used to run regularly*]. The Strong Kleene disjunction only predicts a weaker, conditional presupposition that [$_{\neg\phi}$ *if John does not hate exercise*], [$_p$ *he used to run regularly*]. This is dubbed as *Proviso Problem* by Geurts (1996).

There are several attempts to overcome the proviso problem, which can be classified into two categories. One is to depart from the intuition of Strong Kleene logic and construct a different mechanism for presupposition projection which predicts the non-conditional presupposition for (1a). This line of attempt is formalized as a version of Discourse Representation Theory (DRT; Kamp 1979, 1981; Kamp & Reyle 1993, et seq) by van der Sandt (1992) and Geurts (1996), among others. The other is to accept the weak, conditional presupposition as a consequence of the semantic setup, but to derive the non-conditional, stronger presupposition through an additional apparatus. Fox (2013), for example, suggests strengthening conditional presuppositions to non-conditional presuppositions through pragmatic reasoning. Mayr & Romoli (2016), on the other hand, claims that both conditional and non-conditional presuppositions should be derived semantically. They argue that non-conditional presuppositions arise when a disjunction is interpreted *exclusively*. Their proposal is reviewed in more detail below.

It is worth noting here that the second line of analysis that accepts conditional presupposition as a consequence is supported by the fact that some sentences do have a conditional presupposition, as exemplified by (4).

(4) Either John is not a scuba diver, or he forgot to bring his wet suit.
Presuppose: If John is a scuba diver, he has a wet suit. (Katzir & Singh 2012: 155)

The analysis developed by van der Sandt (1992) and Geurts (1996) does not have any way to predict the conditional presupposition in (4). However, see Schlenker (2011) for an improvement of the DRT-based analysis on this point.

2 The Issue: Presupposition and Exhaustification

In this section, I examine the proposal by Mayr & Romoli (2016) to set up the background for the main concern of this paper. Mayr & Romoli (2016) observes that under the Strong Kleene definition, *exclusive disjunction* $\phi \bar{\vee} \psi_p$ presuppose p. Strong Kleene exclusive disjunction is characterized by the truth table in (5). $\phi \bar{\vee} \psi$ is defined only if both disjuncts are defined. Hence, it projects the presuppositions of the disjuncts unconditionally.

(5) Strong Kleene Exclusive Disjunction

$\bar{\vee}$	1	0	*
1	0	1	*
0	1	0	*
*	*	*	*

If the disjunction *either...or...* is translated into the Strong Kleene exclusive disjunction, the non-conditional presupposition in (1a) is correctly predicted. The translation of *either...or...* into exclusive disjunction is empirically valid as well, because the *either...or...* disjunction is obligatorily interpreted as exclusive (Spector 2014).

Mayr & Romoli (2016) propose to derive exclusive disjunction by *exhaustifying* inclusive disjunction, and adopt the exhaustification operator EXH (Chierchia 2006, a.o.). Applied to disjunction, EXH($\phi \vee \psi$) results in negating a *stronger scalar alternative* of disjunction. Here the relevant stronger alternative is conjunction $\phi \wedge \psi$. Negating the conjunction together with asserting the disjunction results in exclusive disjunction, as in (6).

(6) $\text{EXH}(\phi \vee \psi) = (\phi \vee \psi) \wedge \neg(\phi \wedge \psi)$

Suppose that ψ has a presupposition p. Then $\text{EXH}(\phi \vee \psi_p)$ is equivalent to $(\phi \vee \psi_p) \wedge \neg(\phi \wedge \psi_p)$. With the Strong Kleene definitions of conjunction and negation in (7), this formula ends up presupposing p.

(7)

a.

$\wedge$	1	0	*
1	1	0	*
0	0	0	0
*	*	0	*

b.

$\neg$	
1	0
0	1
*	*

Summing up the proposal by Mayr & Romoli (2016), they observe that exclusive disjunction under the Strong Kleene definition projects the presuppositions of each disjunct, if any. They propose to avoid the proviso problem by appealing to the exclusivity of disjunction.

However, their proposal faces a challenge in analyzing the filtering case in (1b), repeated below. If the disjunction is interpreted exclusively, the proposal wrongly predicts that the presupposition p that *John used to run regularly* is projected.

(1b) Either [$_\phi$ John never used to run regularly] or [$_\psi$ he **stopped** running regularly].
No presupposition

The projection of presuppositions in exclusive disjunction further raises another empirical problem. Notice that, in (1a), it is not only that case that $\neg\phi \rightarrow p$, is it also the case that $\phi \wedge p = \bot$ ($\bot$ for a contradiction). Then p projecting to the entire disjunction entails that ϕ is false. This is a violation of *genuineness*, extensively discussed by (Zimmermann (2000)), stated as (8). A felicitous assertion of disjunction in a natural language requires that each disjunct be *possible*.

(8) *Genuineness*
$\phi \vee \psi$, $\phi \bar{\vee} \psi$ are felicitous only if $\Diamond\phi$ and $\Diamond\psi$.

This issue is real and calls for a solution, especially because, as noted above, the *either...or...* disjunction is always interpreted exclusively. Thus, the problem is more general and robust: as long as we suppose the Strong Kleene definition of logical connectives, the filtering effect observed in *either...or...* disjunction is not predicted, and (1b) necessarily leads to a violation of genuineness.

The current theoretical dilemma we have been facing is summarized as (9).

(9) a. Strong Kleene disjunction faces the proviso problem.

b. Exclusive disjunction avoids the proviso problem.

c. But since *either...or...* is always interpreted exclusively, the filtering effect is not expected, and a violation of genuineness ensues.

The following section is devoted to resolving the dilemma in (9). Before proposing the analysis, in the rest of this section, I show the above dilemma arises even with a more sophisticated theory of exhaustification, which utilizes the notion of *innocent exclusivity* (Fox 2007).

In more elaborated theories of exhaustification after Fox (2007), the operator Exh is defined as (10). $\textsc{Alt}_\phi$ is a set of scaler alternatives of ϕ. $IE(\phi, \textsc{Alt}_\phi)$ defines a set of *innocently excludable* alternatives within $\textsc{Alt}_\phi$, which is defined as (11).

(10) $\textsc{Exh}(\phi) := \phi \wedge \forall \psi \in IE(\phi, \textsc{Alt}_\phi) : \neg\psi$

(11) $IE(\phi, \textsc{Alt}) := \cap \left\{ A \;\middle|\; \begin{array}{l} A \subseteq \textsc{Alt}_\phi \text{ \& } A \text{ is a maximal subset of } \textsc{Alt}_\phi \text{ such that} \\ \{\neg p \mid p \in A\} \cup \phi \text{ is consistent} \end{array} \right\}$,

where a set of propositions is *consistent* if and only if there is a world w in which all the propositions in the set are (defined and) true.

For disjunction $\phi \vee \psi$, which has no presupposition, $\textsc{Alt}_{\phi\vee\psi} = \{\phi\wedge\psi, \phi, \psi\}$. The maximal subsets of $\textsc{Alt}_{\phi\vee\psi}$ that meet the consistency condition is $\{\phi\wedge\psi, \phi\}$ and $\{\phi\wedge\psi, \psi\}$, because $\{\phi\vee\psi, \neg(\phi\wedge\psi), \neg\phi\}$ and $\{\phi\vee\psi, \neg(\phi\wedge\psi), \neg\psi\}$ are consistent. The set of innocently exclusive alternatives is the intersection of these two sets, $\{\phi\wedge\psi, \phi\} \cap \{\phi\wedge\psi, \psi\}$, hence $\{\phi\wedge\psi\}$. The Exh operator applied to the disjunction negates the proposition in the set, which results in $(\phi \vee \psi) \wedge \neg(\phi \wedge \psi)$, exclusive disjunction.

Consider applying Exh defined as (10) to (1b). Recall that in (1b) $\phi \wedge p = \bot$. Keeping this in mind, suppose that $\textsc{Alt}_{\phi\vee\psi_p} = \{\phi, \psi_p, \phi \wedge \psi_p\}$. The set of innocently excludable alternatives is determined as in (12). $\{\phi \vee \psi_p, \neg\psi_p\}$ is never consistent. In order for $\neg\psi_p$ to be true, p must be true and ψ must be false (recall the truth table of negation above). Since $\phi \wedge p = \bot$, in any world where p is true, ϕ is false. There is no world where both propositions in the set are true, which means that ϕ_p is not in the innocently excludable set. On the other hand, $\{\phi \vee \psi_p, \neg\phi\}$ is consistent when ϕ is false and ψ_p is true, and so are $\{\phi \vee \psi_p, \neg(\phi \wedge \psi_p)\}$ and $\{\phi \vee \psi_p, \neg\phi, \neg(\phi \wedge \psi_p)\}$. Therefore, $\{\phi, \phi \wedge \psi_p\}$ is the maximal subset of $\textsc{Alt}_{\phi\vee\psi_p}$ which meets the consistency condition.

(12) a. $\{\phi \vee \psi_p, \neg\psi_p\}$: inconsistent. $p = 1$ for $\neg q_\pi = 1$. By the assumption $\phi \wedge p = \bot$, $p = 0$.

b. $\{\phi \vee \psi_p, \neg\phi\}$: consistent if $\phi = 0$ and $\psi_p = 1$ (hence $p = 1$)

c. $\{\phi \vee \psi_p, \neg(\phi \wedge \psi_p)\}$: consistent if $\phi = 0$ and $\psi_p = 1$.

d. $\{\phi \vee \psi_p, \neg\phi, \neg(\phi \wedge \psi_p)\}$: consistent if $\phi = 0$ and $\psi_\pi = 1$.

Then $\textsc{Exh}(\phi \vee \psi_p)$ will results in (13), which *entails* ψ_p, and therefore, presupposes p. Thus, even under the sophisticated theory of exhaustification, the problem stated in (9c) persists.

(13) $(\phi \vee \psi_p) \wedge \neg\phi \wedge \neg(\phi \wedge \psi_p)$
$\rightsquigarrow \psi_p$

3 Proposal

I propose to resolve the problem by adopting the so-called *weak negation* within the definition of the Exh operator. The weak negation in trivalent logic is defined as (14a). It returns *false* when its prejacent is *undefined*. The strong negation in (14b) is what we have assumed so far.

(14) a. Weak Negation

$\sim$	
1	0
0	1
$*$	1

b. Weak Negation

$\neg$	
1	0
0	1
$*$	$*$

Suppose that the Exh operator, applied to (1b), is defined with the weak negation as in (15). It filters any presupposition induced by the conjunction $(\phi \wedge \psi)$.

(15) $\text{Exh}(\phi \vee \psi) = (\phi \vee \psi) \wedge \sim (\phi \wedge \psi)$

However, (16) should not be *the* definition of the Exh operator because the definition does not predict the non-conditional presupposition observed in (1a), again facing the proviso problem. I thus argue that the Exh operator conflates between the definition with the canonical negation in (16a) and the one with the weak negation in (16b).

(16) a. $\text{Exh}(\phi \vee \psi) = (\phi \vee \psi) \wedge \neg(\phi \wedge \psi)$
b. $\text{Exh}(\phi \vee \psi) = (\phi \vee \psi) \wedge \sim (\phi \wedge \psi)$

The conflation should be properly constrained. Otherwise, the proposal makes no falsifiable prediction. I argue that the Exh operator defined as (16b) is felicitously used only when the use of (16a) leads to a violation of a pragmatic principle.

Recall that natural language disjunction $\phi \vee \psi$ is felicitous only when each disjunct is possibly true, a principle called *genuineness*. Recall further that when $\phi \wedge p = \bot$ in $\phi \vee \psi_p$, exhaustifying the disjunction following the definition in (16a) necessarily leads to a violation of genuineness because the resultant exclusive disjunction presupposes p. Therefore, the definition in (16b) must be used, canceling the unwanted non-conditional presupposition.

3.1 Semi-Formalizing the constraint

The weak negation is also used, especially as an extra-clausal negation, when the speaker wants to cancel the presupposition of a sentence explicitly. Consider, for example the discourse in (17). If the negation *it is not the case* is translated as the strong negation $\neg$, the discourse ends up being contradictory. The first sentence presupposes that there is a king in France, but the second sentence negates the truth of the presupposition. Suppose that the two sentences are coordinated by conjunction. Then the conjunction is either *undefined* (the second conjunct is true, and the first sentence is undefined), or *false* (the first sentence is defined (true or false) and the second sentence is false).

(17) *It is not the case* that the King of France is bald! France does not have a king (in the first place).

I argue that the weak negation is used in sentences like (17) or in the course of exhaustification if sentences cannot turn out to be true otherwise. In (17), for example, unless the negation is translated as the weak negation $\sim$ the discourse cannot be true.

To extend this proposal to the case of exhaustification in (1b), I follow Goldstein (2019) and assume that the genuineness condition is also a presupposition of disjunction. More specifically:

(18) $\phi \vee \psi$ *presupposes* $\Diamond\phi \wedge \Diamond\psi$

Then consider the crucial case $\phi \vee \psi_p$, where $\phi \wedge p = \bot$. By applying exhaustification, the disjunction results in exclusive disjunction. Suppose first that exhaustification is achieved with the strong negation $\neg$. It has two presuppositions, p and $\Diamond\phi \wedge \Diamond\psi$. But since $\phi \wedge p = \bot$, ϕ is no longer possibly true. Therefore, the formula is necessarily undefined.[1]

(19) $(\phi \vee \psi_p) \wedge \neg(\phi \wedge \psi_p)$
$\rightsquigarrow \psi_p$
$\rightsquigarrow \Diamond\phi \wedge \Diamond\psi_q$

If the weak negation is employed instead, the non-conditional presupposition disappears. The genuineness condition can be satisfied without any problem, and the formula can turn out to be true.

(20) $(\phi \vee \psi_p) \wedge \sim (\phi \wedge \psi_p)$
$\rightsquigarrow \Diamond\phi \wedge \Diamond\psi_q$

Therefore, I propose to constrain the use of the weak negation by the constraint in (21).

(21) Suppose that $\phi_\neg$ is a formula with a strong negation, $\phi_\sim$ is its counterpart with a weak negation. $\phi_\sim$ is felicitously asserted only if $\phi_\neg$ is necessarily *undefined* or *false*.

4 Conclusion

In this paper, I pointed out a robust problem raised by the interplay of obligatory strengthening of a disjunction to exclusive disjunction and presupposition projection. In order to avoid a violation of genuineness, or undefinedness, I have proposed that the exhaustification operator should at least sometimes employ the weak negation, which is inserted as a last resort.

The current proposal can further be formalized under the *Floating-$\mathcal{A}$* theory (Beaver & Krahmer 2001), which defines the weak negation as $\neg\mathcal{A}$, where $\mathcal{A}$ is an assertion operator – $\mathcal{A}\phi$ is true if ϕ is true, and false if ϕ is false or undefined. If such re-formalization is achieved, the theory would be more general, but I leave it for future work.

1. The proposal is best formalized under *Update Semantics* (Veltman 1996, Groenendijk et al. (1996)), or *Team-based semantics* (Aloni 2022). The limitation of space prevents me from laying out the exact technicality.

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References

Aloni, M. (2022). Logic and conversation: The case of free choice. *Semantics and Pragmatics*, *15*(5).

Beaver, D. & Krahmer, E. (2001). A partial account of presupposition projection. *Journal of Logic, Language, and Information*, *10*, 147–182.

Chierchia, G. (2006). Broaden Your Views: Implicatures of Domain Widening and the "Logicality" of Language. *Linguistic Inquiry*, *37*(4), 535–590. Publisher: The MIT Press.

Fox, D. (2007). Free choice disjunction and the theory of scalar implicature. In U. Sauerland & P. Stateva (Eds.), *Presupposition and Implicature in Compositional Semantics* (pp. 71–120). New York, NY: Palgrave Macmillan.

Fox, D. (2013). Presupposition projection from quantificational sentences: Trivalence, local accommodation, and presupposition strengthening. In I. Caponigro & C. Cecchetto (Eds.), *From Grammar to Meaning* (pp. 201–232). Cambridge University Press.

Geurts, B. (1996). Local satisfaction guaranteed: A presupposition theory and its problems. *Linguistics and Philosophy*, *19*(3), 259–294.

Goldstein, S. (2019). Free choice and homogeneity. *Semantics and Pragmatics*, *12*(23), 1–47.

Groenendijk, J., Stokhofand, M., & Veltman, F. (1996). Coreference and modality. In S. Lappin (Ed.), *Handbook of Contemporary Semantic Theory* (pp. 179–216). Blackwell.

Kamp, H. (1979). Events, Instants and Temporal Reference. In *Semantics from Different Point of View (Springer Series in Language and Commnication 6)* (pp. 376–417). Berlin: Springer.

Kamp, H. (1981). A Theory of Truth and Semantic Representation. In Groenendijk, J., Janssen, T., & Stokhof, M. (Eds.), *Formal methods in the Study of Language*, (pp. 189–222)., Amsterdam.

Kamp, H. & Reyle, U. (1993). *From Discourse to Logic*. Dordrecht: Kluwer.

Karttunen, L. (1973). Presuppositions of compound sentences. *Linguistic Inquiry*, *4*, 169–193.

Katzir, R. & Singh, R. (2012). Two restrictions on possible connectives. In *Theories of Everything In Honor of Ed Keenan* (pp. 154–152). UCLA Working Papers in Linguistics, Theories of Everything.

Mayr, C. & Romoli, J. (2016). Satisfied or exhaustified: an ambiguity account of the proviso problem. In *Semantics and Linguistic Theory*, volume 26, (pp. 892). Linguistic Society of America.

Schlenker, P. (2011). DRT with local contexts. *Natural Language Semantics*, *19*(4), 373–392.

Spector, B. (2014). Global positive polarity items and obligatory exhaustivity. *Semantics and Pragmatics*, *7*.

Spector, B. & Sudo, Y. (2017). Presupposed ignorance and exhaustification: how scalar implicatures and presuppositions interact. *Linguistics and Philosophy*, *40*(5), 473–517.

van der Sandt, R. A. (1992). Presupposition projection and anaphora resolution. *Journal of Semantics*, 333–377.

Veltman, F. (1996). Defaults in update semantics. *Journal of Philosophical Logic*, *25*(3), 221–261.

Zimmermann, T. E. (2000). Free choice disjunction and epistemic modality. *Natural Language Semantics*, *8*(4), 255–290.